In an arbitrary cartesian coordinate system [see Fig. 2-2(*b*)],

$$\mathbf{E} = \frac{Q}{4\pi\epsilon_0 R^2}\mathbf{a}_R$$

The units of **E** are newtons per coulomb (N/C) or the equivalent, volts per meter (V/m).

EXAMPLE 2. Find **E** at (0, 3, 4) m in cartesian coordinates due to a point charge $Q = 0.5\ \mu\text{C}$ at the origin.
In this case

$$\mathbf{R} = 3\mathbf{a}_y + 4\mathbf{a}_z \qquad R = 5 \qquad \mathbf{a}_R = 0.6\mathbf{a}_y + 0.8\mathbf{a}_z$$

$$\mathbf{E} = \frac{0.5 \times 10^{-6}}{4\pi(10^{-9}/36\pi)(5)^2}(0.6\mathbf{a}_y + 0.8\mathbf{a}_z)$$

Thus $|\mathbf{E}| = 180\ \text{V/m}$ in the direction $\mathbf{a}_R = 0.6\mathbf{a}_y + 0.8\,\mathbf{a}_z$.

2.3 CHARGE DISTRIBUTIONS

Volume Charge

When charge is distributed throughout a specified volume, each charge element contributes to the electric field at an external point. A summation or integration is then required to obtain the total electric field. Even though electric charge in its smallest division is found to be an electron or proton, it is useful to consider continuous (in fact, differentiable) charge distributions and to define a *charge density* by

$$\rho = \frac{dQ}{dv} \qquad (\text{C/m}^3)$$

Note the units in parentheses, which is meant to signify that ρ will be in C/m^3 provided that the variables are expressed in proper SI units (C for Q and m^3 for v). This convention will be used throughout this book.

With reference to volume v in Fig. 2-3, each differential charge dQ produces a differential electric field

$$d\mathbf{E} = \frac{dQ}{4\pi\epsilon_0 R^2}\mathbf{a}_R$$

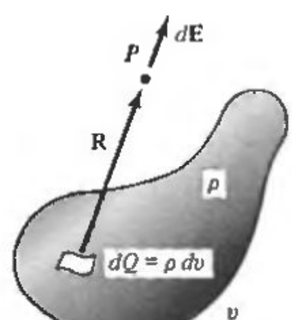

Fig. 2-3

at the observation point P. Assuming that the only charge in the region is contained within the volume, the total electric field at P is obtained by integration over the volume:

$$\mathbf{E} = \int_v \frac{\rho\mathbf{a}_R}{4\pi\epsilon_0 R^2}\,dv$$

Sheet Charge

Charge may also be distributed over a surface or a sheet. Then each differential charge dQ on the sheet results in a differential electric field

$$d\mathbf{E} = \frac{dQ}{4\pi\epsilon_0 R^2}\mathbf{a}_R$$

at point P (see Fig. 2-4). If the *surface charge density* is ρ_s (C/m^2) and if no other charge is present in the region, then the total electric field at P is

$$\mathbf{E} = \int_S \frac{\rho_s \mathbf{a}_R}{4\pi\epsilon_0 R^2}\, dS$$

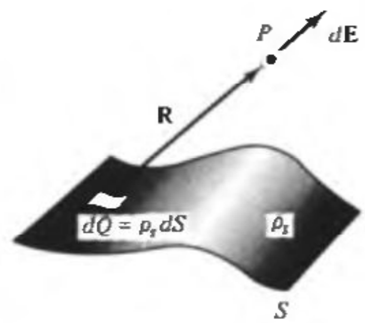

Fig. 2-4

Line Charge

If charge is distributed over a (curved) line, each differential charge dQ along the line produces a differential electric field

$$d\mathbf{E} = \frac{dQ}{4\pi\epsilon_0 R^2}\mathbf{a}_R$$

at P (see Fig. 2-5). And if the *line charge density* is ρ_ℓ (C/m), and no other charge is in the region, then the total electric field at P is

$$\mathbf{E} = \int_L \frac{\rho_\ell \mathbf{a}_R}{4\pi\epsilon_0 R^2}\, d\ell$$

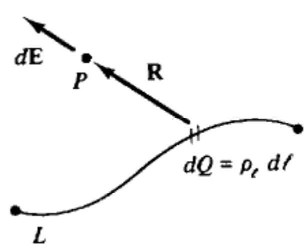

Fig. 2-5

It should be emphasized that in all three of the above charge distributions and corresponding integrals for $\mathbf{E}$, the unit vector $\mathbf{a}_R$ is variable, depending on the coordinates of the charge element dQ. Thus $\mathbf{a}_R$ cannot be removed from the integrand. It should also be noticed that whenever the appropriate integral converges, it defines $\mathbf{E}$ at an *internal* point of the charge distribution.